Principles of modern fault diagnosis

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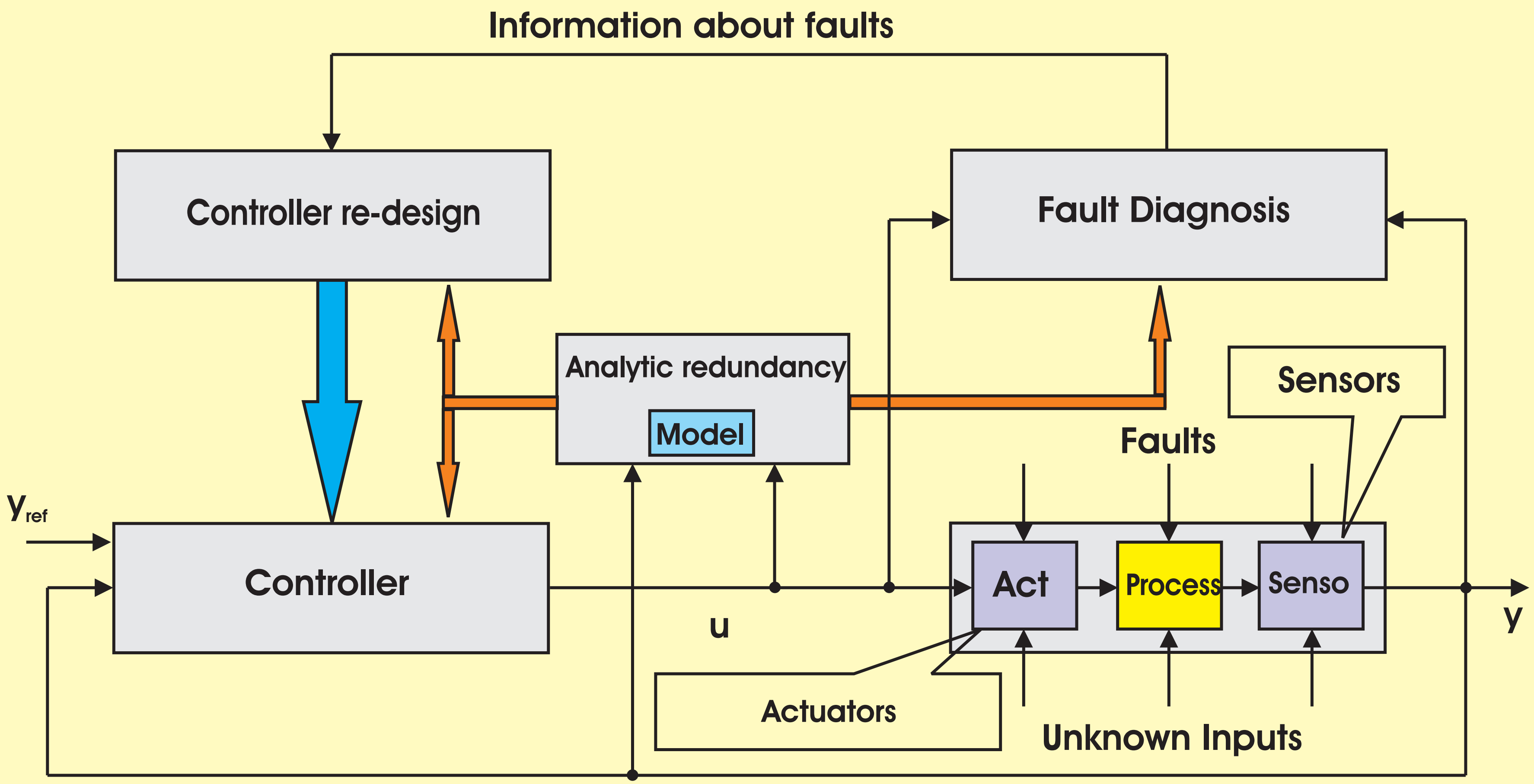
OUTLINE

- Fundamental concepts, definitions and history of fault diagnosis
- Classical analytical approaches to residual generation
- Classical analytical approaches to residual evaluation
- Non-linear extensions of classical techniques
- Towards robustness – active and passive approaches
- Tackling nonlinearities and robustness problem
- Issues of analytical techniques = challenges for soft computing

☛ FUNDAMENTAL CONCEPTS, DEFINITIONS AND HISTORY OF FAULT DIAGNOSIS

➜ Modern control system with FDI



➜ Fundamental definitions

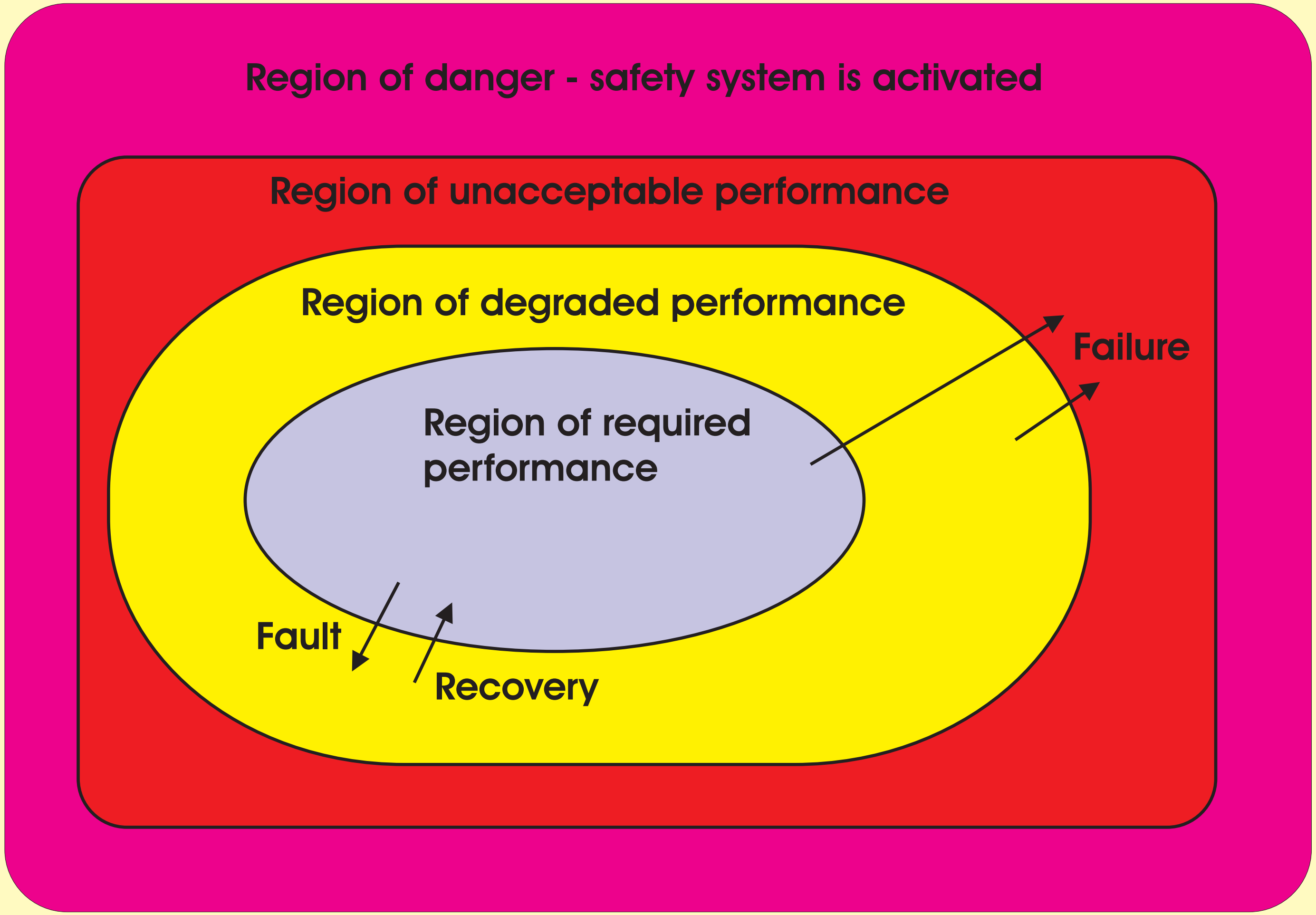
Fault: an unpermitted deviation of at least one characteristic property or parameter of the system from the normal condition

Failure: a permanent interruption of the system ability to perform a required function under specified operating conditions

Symptom: a change of an observable quantity from normal behaviour

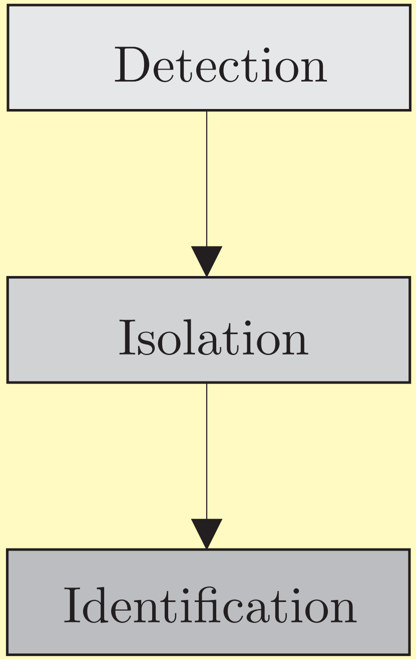
Fault diagnosis: the determination of the kind, size, location and occurrence time of a fault

➜ Regions of required and degraded system performance



Safety: describes the absence of danger. A safety system is a part of control equipment that protects a technological system from a permanent damage.

➜ Diagnostic steps

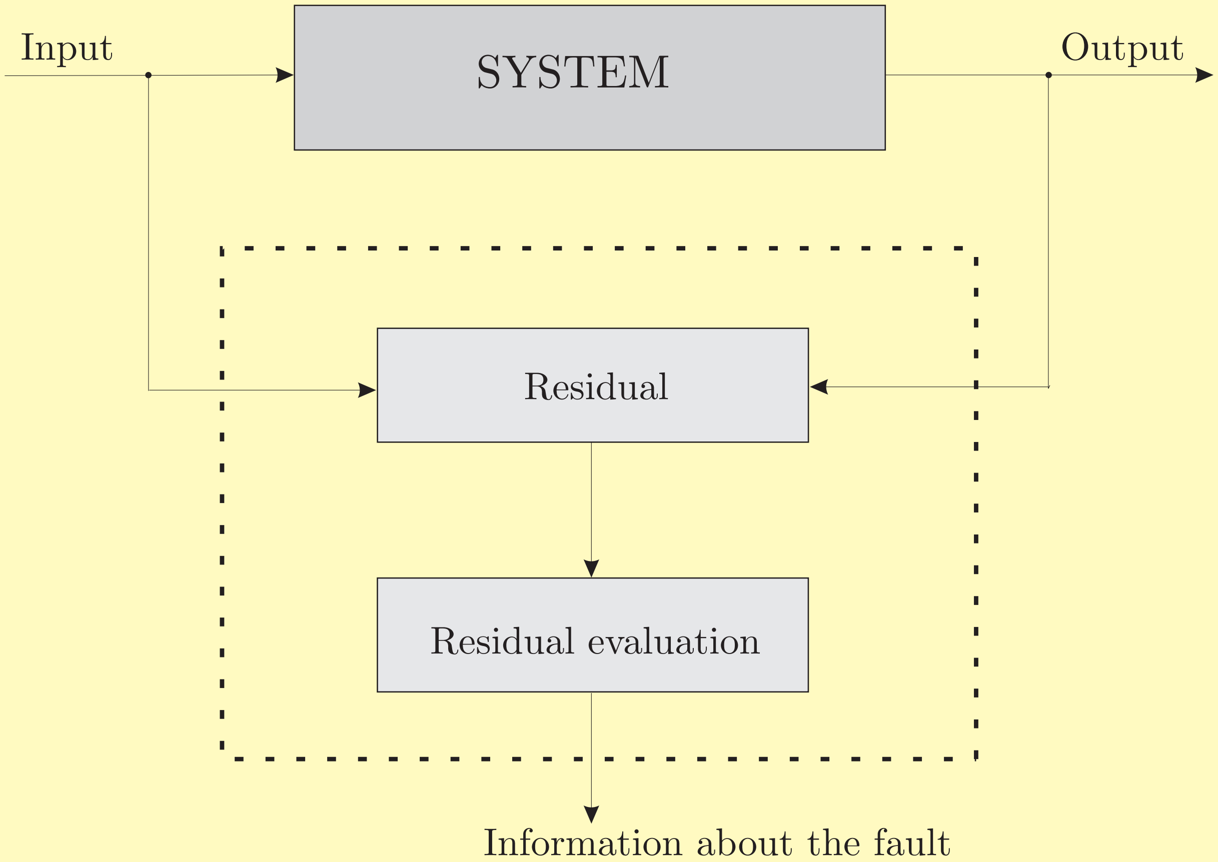


Fault detection: the determination of faults (and their detection time) present in the system

Fault isolation: the determination of the kind and location of a fault

Fault identification: the determination of the size and time-variant behaviour of a fault

➜ Fault diagnosis as a two-step procedure



Residual: a fault indicator obtained with a deviation between measurements and model-based computations

➜ Fault isolation schemes

Dedicated scheme: A set of residuals is generated where each residual is sensitive to one fault only. The diagnostic logic boils down to

$$r_{i,k} > T_i \Rightarrow f_{i,k} \neq 0, \quad i = 1, \ldots, g,$$

where $\boldsymbol{r}_k \in \mathbb{R}^g$ stands for the residual vector, T_i denotes the threshold, $\boldsymbol{f}_k \in \mathbb{R}^g$ is the fault vector.

Generalized scheme: A set of residuals is generated where each residual is sensitive to all but one fault. The diagnostic logic boils down to

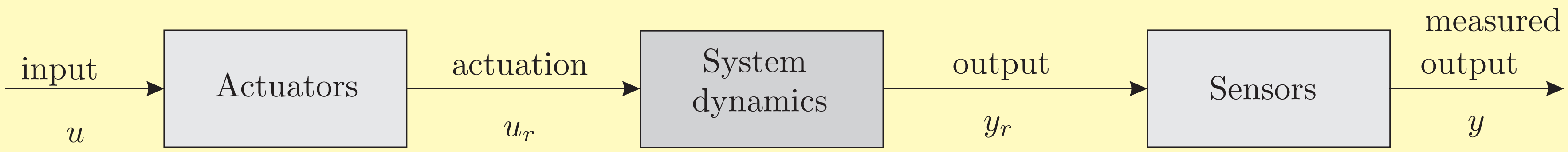
$$\left.\begin{array}{l} r_{i,k} \leqslant T_i \\ r_{j,k} > T_j, j = 1 \ldots, i-1, i+1, \ldots, g \end{array}\right\} \Rightarrow f_{i,k} \neq 0, \quad i = 1, \ldots, g.$$

➜ Detectability and isolability

Detectability: The i-th fault $f_{i,k}$ is detectable if there exists a stable residual generator such that $\boldsymbol{r}_k$ is affected by $f_{i,k}$.

Isolability: The i-th fault $f_{i,k}$ is isolable if there exists a stable residual generator such that the fault $f_{i,k}$ is distinguishable from other faults based on $\boldsymbol{r}_k$.

➜ Classification of faults



Actuators faults: can be viewed as any malfunction of equipment that actuates the system, e.g. a malfunction of an electro-mechanical actuator for a diesel engine

System dynamics faults (or component faults): occur when some changes in the system make the dynamic relation invalid, e.g. a leak in a tank in a two-tank system

Sensors faults: can be perceived as serious measurement variations

➜ Outline of the model-based FDI history

1971: Fault detection for linear dynamic systems

Beard: PhD thesis, MIT

1975: Development of observer-based techniques

Clark *et al.*: IEEE Trans. Aero. and Electron.

1979: Development of parity relation methods

Mironovski: Aut. Remote Contr.

1980: Definition of a two-stage diagnostic procedure

Chow and Willsky: Proc. Conf. on Decision and Contr., CDC

1981: Tackling the robustness problem in FDI

Frank and Keller: IEEE Trans. Aero. & Electron. Syst.

1986: Development of FDI for non-linear systems

Hengy and Frank: IFAC Workshop on Fault Detection and Safety in Chemical Plants

➜ Outline of the model-based FDI history

1988: Development of adaptive-threshold-based techniques

Emami-Naeini *et al.*: IEEE Trans. Automat. Contr.

1989: Application of soft computing techniques for FDI

Watanabe *et al.*: AICHE J.

1991: Tackling the robustness problem in FDI for non-linear systems

Frank and Seliger: Control and Dynamic Syst.

Establishment of the IFAC Technical Committee: *Fault Detection, Safety and Supervision of Technical Processes, SAFEPROCESS*

Founder and first Chairman: Prof. Rolf Isermann

...: Further improvements of the existing FDI techniques

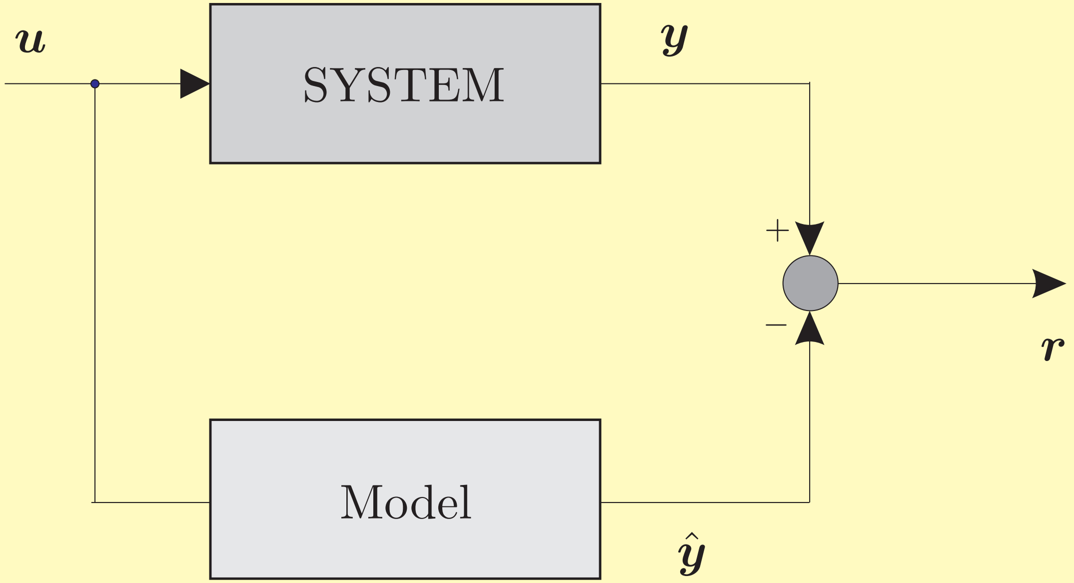
2002: Development of robust soft computing techniques for FDI

Witczak, Korbicz, *et al.*

☛ CLASSICAL ANALYTICAL APPROACHES TO RESIDUAL GENERATION

- Direct-model-based residual generation scheme
- Parameter-estimation-based techniques: Bakiotis, Raymond and Rault (1979): Proc. IEEE Conf. on Decision and Control, CDC
- Parity relation residual generation schemes: Mironovski (1979): Aut. Remote Contr., Vol. 40
- Observer-based residual generators: Clark, Fosth and Walton (1975): IEEE Trans. Aero. and Electron. Syst., Vol. 11

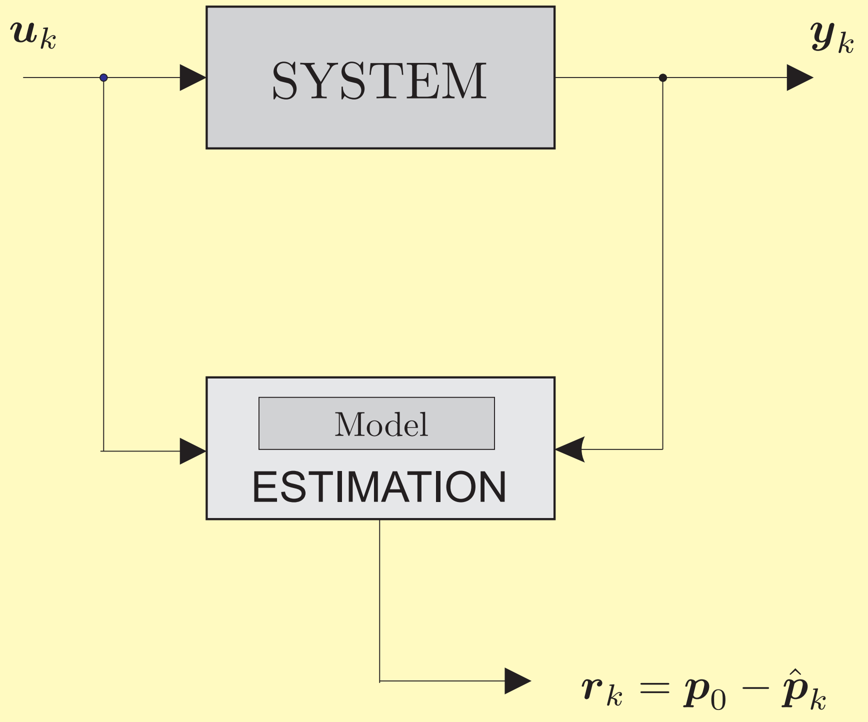
➜ Direct-model-based residual generation scheme



Residual:

$$r_k = y_k - \hat{y}_k$$

➜ Parameter-estimation-based techniques



Residual:

$$\boldsymbol{r}_k = \boldsymbol{p}_0 - \hat{\boldsymbol{p}}_k,$$

where $\boldsymbol{p}_0$ stands for the nominal (non-faulty) parameter vector and $\hat{\boldsymbol{p}}_k$ is the parameter estimate

➜ Parameter-estimation-based techniques

Model structure:

$$y_k = \boldsymbol{g}(\boldsymbol{\phi}_k, \boldsymbol{p}_k),$$

where $\boldsymbol{\phi}_k$ may contain the previous or current system input $\boldsymbol{u}_k$, the previous system or model output (y or $\hat{y}$), and the previous prediction error.

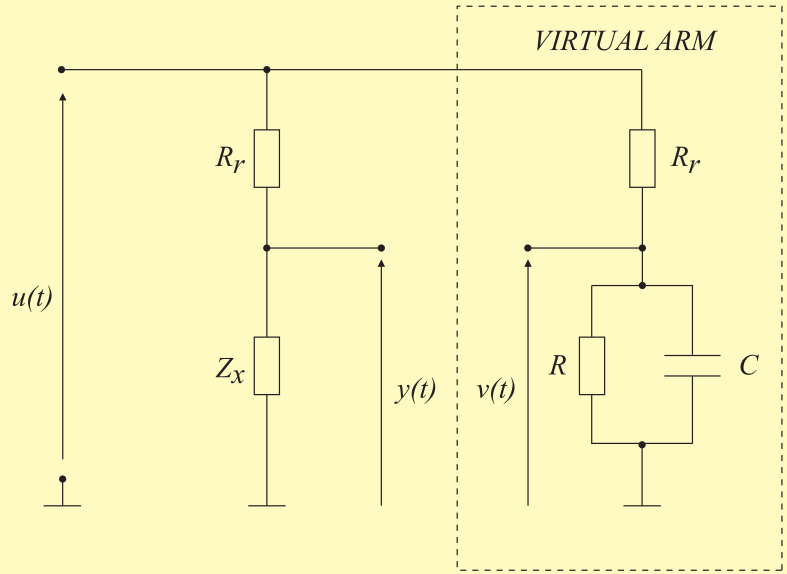
Main assumptions:

- The model structure $\boldsymbol{g}(\cdot)$ is assumed to be linear with respect to the parameters $\boldsymbol{p}_k$
- the model parameters should have physical meaning, i.e. they should correspond to the parameters of the system

➜ Parameter-estimation-based techniques – an illustrative example

- Different physical quantities (force, pressure, displacement, etc.) can be transduced into impedance values
- *Problem:* in order to measure and diagnose these quantities it is necessary to develop an accurate on-line impedance measurement method
- *Proposed solution:* **a virtual bridge**

  (L. Angrisani *et al.* (1996): IEEE Trans. Instrument. and Measurement, Vol. 45, No. 6)



➜ *Task:* to obtain R and C based on the measurements of $u(t)$ and $v(t)$

- Current equality:

$$C\frac{dv(t)}{dt} + \frac{v(t)}{R} = \frac{u(t)-v(t)}{R_r}.$$

Assuming that $u(t) = U\sqrt{2}\sin(\omega t)$, the steady-state solution can be written as

$$v(t) = \rho U\sqrt{2}R((R+Rr)\sin(\omega t) - R_r RC\omega\cos(\omega t)),$$

where $\rho = \left(R^2 + 2R_rR + R_r^2(1+\omega^2R^2C^2)\right)^{-1}$.

- Discrete-time form:

$$v_k = p_1u_{1,k} + p_2u_{2,k},$$

where

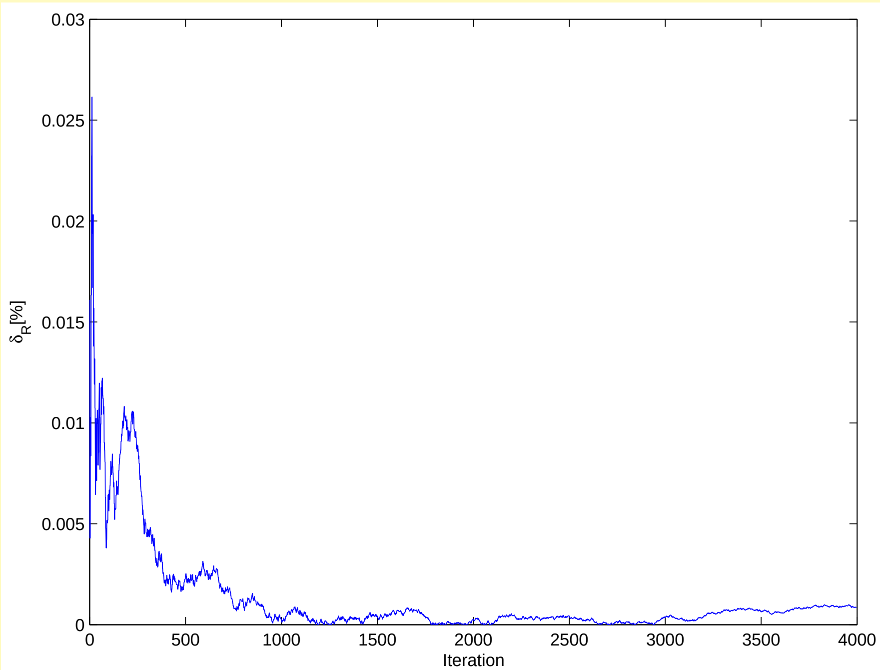
$$p_1 = \rho R(R+R_r), \quad p_2 = \rho R_r C\omega R^2$$

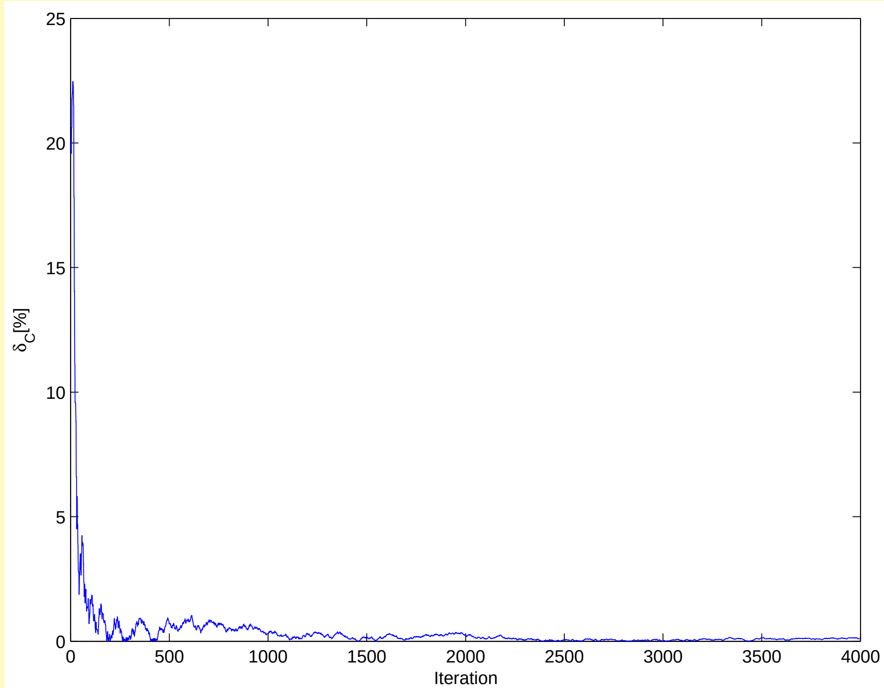
$u_{1,k} = U\sqrt{2}\sin(\omega k\tau)$, $u_{2,k} = U\sqrt{2}\cos(\omega k\tau)$, where τ is the sampling time

- Since v_k is non-linear with respect to R and C (L. Angrisani *et al.* (1996): IEEE Trans. Instrument. and Measurement, Vol. 45, No. 6) proposed to estimate them with a non-linear optimization technique.
- Is it really necessary to use non-linear parameter estimation techniques for estimating R and C?
- We propose to estimate p_1 and p_2 with the classical recursive least-square algorithm and then to obtain the estimates of R and C according to (Witczak (2005): IFAC World Congress):

$$\hat{R} = -\frac{R_r(\hat{p}_1^2 + \hat{p}_2^2)}{\hat{p}_1^2 + \hat{p}_2^2 - \hat{p}_1}, \quad \hat{C} = -\frac{\hat{p}_2}{R_r\omega(\hat{p}_1^2 + \hat{p}_2^2)}$$

➜ Exemplary run of the proposed algorithm





➜ Advantages and drawbacks of parameter-estimation-based residual generation techniques

- The model has to be linear with respect to the parameters
- The detection of faults in sensors and actuators is possible but rather complicated, i.e. a suitable transformation of parameter deviations into these faults has to be determined
- The detection and isolation of parametric faults are very straightforward
- A variety of on-line parameter estimation methods can be applied

➜ Parity relation

The basic idea underlying the parity relation approach is to check the consistency of the system measurements

$$\boldsymbol{y}_k = \boldsymbol{C}\boldsymbol{x}_k + \boldsymbol{f}_k + \boldsymbol{v}_k,$$

where $\boldsymbol{y}_k \in \mathbb{R}^m$ is the measured output, and $\boldsymbol{x}_k \in \mathbb{R}^n$ is the state, $\boldsymbol{v}_k$ is the noise, and $\boldsymbol{f}_k$ stands for the sensor faults.
The measurement vector $\boldsymbol{y}_k$ can be combined into a set of linearly independent parity equations, i.e.

$$\boldsymbol{r}_k = \boldsymbol{V}\boldsymbol{y}_k = \boldsymbol{V}\left[\boldsymbol{C}\boldsymbol{x}_k + \boldsymbol{f}_k + \boldsymbol{v}_k\right].$$

Assumptions:

- n signals are measured with m sensors, where $m > n$
- $\mathrm{rank}(\boldsymbol{C}) = n$

➜ Parity relation

Design procedure:

Set

$$\boldsymbol{V}\boldsymbol{C} = \mathbf{0}$$

to get

$$\boldsymbol{r}_k = \boldsymbol{V}[\boldsymbol{f}_k + \boldsymbol{v}_k].$$

Note that the residual is affected by faults and noise only

Main drawbacks:

- it requires additional hardware, i.e. sensors, which may lead to a significant increase in the cost
- it is useless when $\mathrm{rank}(C) = m < n$

➜ Parity relation with an analytical redundancy

Assumptions:

- analytical redundancy is performed by collecting sensor outputs in a data window, i.e. $\{\boldsymbol{y}_{k-i}\}_{i=0}^{s}$
- since redundancy is related to time, such an approach requires the knowledge of a dynamic model, which can be given as follows:

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k + \boldsymbol{L}_1\boldsymbol{f}_k$$
$$\boldsymbol{y}_k = \boldsymbol{C}\boldsymbol{x}_k + \boldsymbol{D}\boldsymbol{u}_k + \boldsymbol{L}_2\boldsymbol{f}_k.$$

Redundancy relation

$$\underbrace{\begin{bmatrix} \boldsymbol{y}_{k-s} \\ \boldsymbol{y}_{k-s+1} \\ \vdots \\ \boldsymbol{y}_k \end{bmatrix}}_{\boldsymbol{Y}_k} = \boldsymbol{H}\underbrace{\begin{bmatrix} \boldsymbol{u}_{k-s} \\ \boldsymbol{u}_{k-s+1} \\ \vdots \\ \boldsymbol{u}_k \end{bmatrix}}_{\boldsymbol{U}_k} = \boldsymbol{W}\boldsymbol{x}_{k-s} + \boldsymbol{M}\underbrace{\begin{bmatrix} \boldsymbol{f}_{k-s} \\ \boldsymbol{f}_{k-s+1} \\ \vdots \\ \boldsymbol{f}_k \end{bmatrix}}_{\boldsymbol{F}_k}$$

➜ Parity relation with an analytical redundancy

Residual:

$$\boldsymbol{r}_k = \boldsymbol{V}[\boldsymbol{Y}_k - \boldsymbol{H}\boldsymbol{U}_k] = \boldsymbol{V}\boldsymbol{W}\boldsymbol{x}_{k-s} + \boldsymbol{V}\boldsymbol{M}\boldsymbol{F}_k.$$

Design procedure

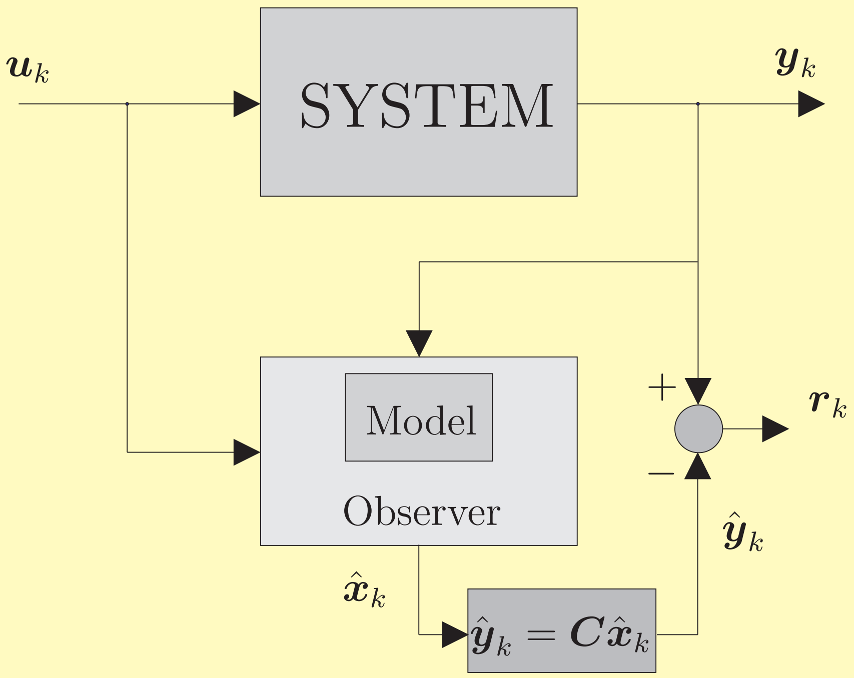
Set $\boldsymbol{V}\boldsymbol{W} = \mathbf{0}$ under $\boldsymbol{V}\boldsymbol{M} \neq \mathbf{0}$.

How to determine the size of the time window?

See (Chen and Patton, 1999) for a comprehensive discussion regarding the size s of the time window.

Main advantage: It can be used for designing a set of residuals that can be applied to sensors and actuators FDI

➜ Observer-based residual generators



System description:

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}_k\boldsymbol{x}_k + \boldsymbol{B}_k\boldsymbol{u}_k + \boldsymbol{L}_{1,k}\boldsymbol{f}_k + \boldsymbol{w}_k$$
$$\boldsymbol{y}_k = \boldsymbol{C}_k\boldsymbol{x}_k + \boldsymbol{D}_k\boldsymbol{u}_k + \boldsymbol{L}_{2,k}\boldsymbol{f}_k + \boldsymbol{v}_k.$$

➜ Luenberger observers and Kalman filters

Deterministic systems – Luenberger observer:

$$\hat{\boldsymbol{x}}_{k+1} = \boldsymbol{A}_k\hat{\boldsymbol{x}}_k + \boldsymbol{B}_k\boldsymbol{u}_k + \boldsymbol{K}_{k+1}(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_k)$$

$$\begin{aligned}\boldsymbol{r}_{k+1} =& \boldsymbol{C}_{k+1}[\boldsymbol{A}_k - \boldsymbol{K}_{k+1}\boldsymbol{C}_k][\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k] + \boldsymbol{C}_{k+1}\boldsymbol{L}_{1,k}\boldsymbol{f}_k\\ &-\boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}\boldsymbol{L}_{2,k}\boldsymbol{f}_k + \boldsymbol{L}_{2,k+1}\boldsymbol{f}_{k+1}\end{aligned}$$

Stochastic systems – Kalman filter:

$$\begin{aligned}\hat{\boldsymbol{x}}_{k+1/k} &= \boldsymbol{A}_k\hat{\boldsymbol{x}}_k + \boldsymbol{B}_k\boldsymbol{u}_k\\ \hat{\boldsymbol{x}}_{k+1} &= \hat{\boldsymbol{x}}_{k+1/k} + \boldsymbol{K}_{k+1}[\boldsymbol{y}_{k+1} - \boldsymbol{C}_{k+1}\hat{\boldsymbol{x}}_{k+1/k}]\end{aligned}$$

$$\begin{aligned}\boldsymbol{r}_{k+1} =& \boldsymbol{C}_{k+1}\boldsymbol{Z}_{k+1}\boldsymbol{A}_k[\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k] + \boldsymbol{C}_{k+1}\boldsymbol{Z}_{k+1}\boldsymbol{L}_{1,k}\boldsymbol{f}_k\\ &+\boldsymbol{M}_{k+1}\boldsymbol{L}_{2,k}\boldsymbol{f}_{k+1} + \boldsymbol{C}_{k+1}\boldsymbol{Z}_{k+1}\boldsymbol{w}_k + \boldsymbol{M}_{k+1}\boldsymbol{v}_{k+1},\end{aligned}$$

where $\boldsymbol{Z}_{k+1} = [\boldsymbol{I} - \boldsymbol{K}_{k+1}\boldsymbol{C}_{k+1}]$ and $\boldsymbol{M}_{k+1} = [\boldsymbol{I} - \boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}]$.

☛ CLASSICAL ANALYTICAL APPROACHES TO RESIDUAL EVALUATION

Deterministic approaches – a fixed threshold

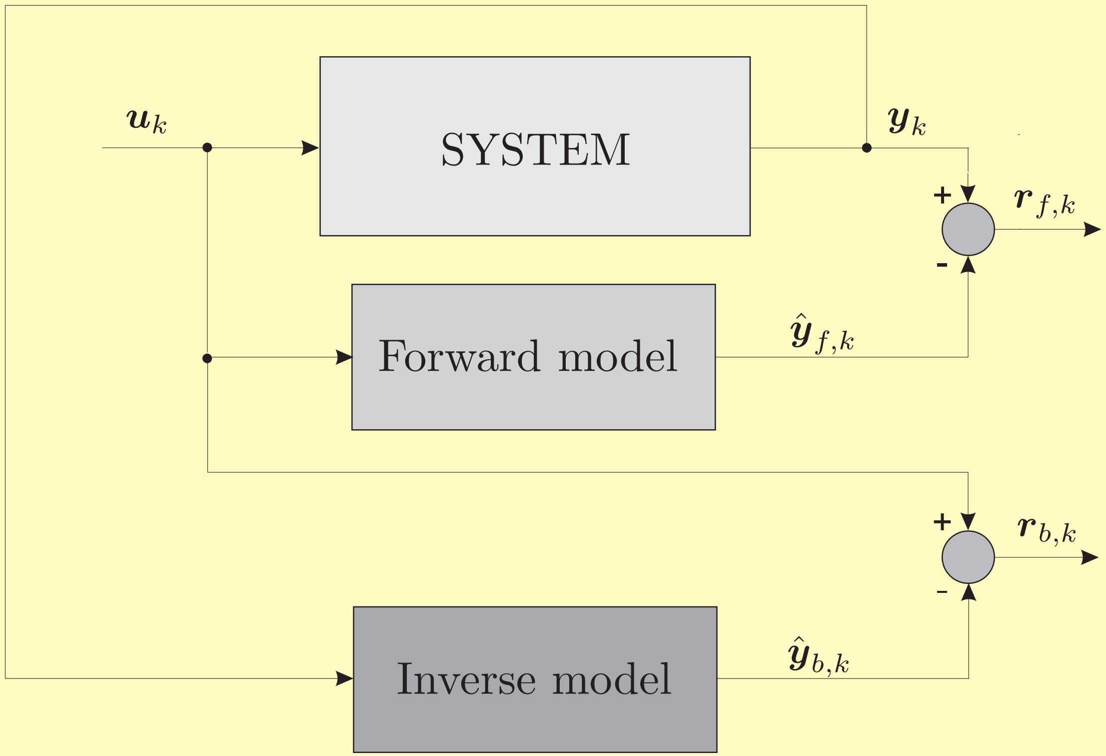
Stochastic approaches

- weighted sum-squared residual testing: Willsky *et al.* (1975): J. Spacecrafts and Rockets, Vol. 12, No. 7
- χ^2 testing: Willsky (1976): Automatica, Vol. 12, No. 7
- sequential probability ration testing: Willsky (1976): Automatica, Vol. 12, No. 7
- generalized likelihood ration testing: Willsky and Jones (1974): IEEE Conf. on Decision and Control, CDC
- cumulative sum algorithm: Nikiforov *et al.* (1993): Automatica
- multiple hypothesis testing: Bogh *et al.* (1995): Contr. Eng. Practice, Vol. 3, No. 12

NON-LINEAR EXTENSIONS OF CLASSICAL TECHNIQUES

Non-linear extensions of parity relation

- Parity relation for polynomial systems: Guernez *et al.* (1997)
- Parity relation for bilinear systems: Shields *et al.* (1997)
- General scheme for non-linear systems: Krishnaswami and Rizzoni *et al.* (1994)



➜ Observers for non-linear systems – deterministic systems

- Extended Luenberger and Kalman observer: Boutayeb and Aubry (1999)
- Observers for Lipschitz non-linear systems: Thau (1973), Witczak (2005):

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k + \boldsymbol{g}(\boldsymbol{x}_k, \boldsymbol{u}_k) + \boldsymbol{L}_{1,k}\boldsymbol{f}_k$$
$$\boldsymbol{y}_{k+1} = \boldsymbol{C}\boldsymbol{x}_{k+1} + \boldsymbol{L}_{2,k+1}\boldsymbol{f}_{k+1},$$

  where $\|\boldsymbol{g}(\boldsymbol{x}_1, \boldsymbol{u}^*) - \boldsymbol{g}(\boldsymbol{x}_2, \boldsymbol{u}^*)\|_2 \leqslant \gamma\|\boldsymbol{x}_1 - \boldsymbol{x}_2\|_2$ and $\gamma > 0$ stands for the Lipschitz constant

- Observers for polynomial and binomial systems: Shields (1997)
- Coordinate transformation: Califano *et al.* (2003): Sys. & Cont. Lett.:

$$\boldsymbol{x}_{k+1} = \boldsymbol{g}(\boldsymbol{x}_k, \boldsymbol{u}_k)$$
$$y_{k+1} = \boldsymbol{h}(\boldsymbol{x}_{k+1})$$

  Transformations $\boldsymbol{z} = \phi(\boldsymbol{x})$ and $\bar{y} = \varphi(y)$ yield

$$\boldsymbol{z}_{k+1} = \boldsymbol{A}(\boldsymbol{u}_k)\boldsymbol{z}_k + \xi(y_k, \boldsymbol{u}_k)$$
$$\bar{y}_{k+1} = \varphi(y_{k+1}) = \boldsymbol{C}\boldsymbol{z}_{k+1}.$$

➜ Observers for non-linear systems

Observers for stochastic non-linear systems

- Extended Kalman filter: Korbicz *et al.*(2004)
- Second-order extended Kalman filter (possible to use)
- Iterated extended Kalman filter (possible to use)
- Particle filter: Hutten and Dearden (2003)

☛ TOWARDS ROBUSTNESS – ACTIVE AND PASSIVE APPROACHES

Main drawback of conventional techniques

lack of robustness to model uncertainty

- Active approaches – the elimination of model uncertainty from the residual:
  - unknown input observers
  - parity relations methods
- Passive approaches – provide an adaptive threshold taking into account model uncertainty:
  - approaches for linear systems
  - linearization-based extensions for non-linear systems

➜ Unknown Input Observer (UIO)

$$
\begin{aligned}
\boldsymbol{x}_{k+1} &= \boldsymbol{A}_k \boldsymbol{x}_k + \boldsymbol{B}_k \boldsymbol{u}_k + \boldsymbol{E}_k \boldsymbol{d}_k + \boldsymbol{w}_k \\
\boldsymbol{y}_k &= \boldsymbol{C}_k \boldsymbol{x}_k + \boldsymbol{v}_k,
\end{aligned}
$$

where

- $\boldsymbol{x}_k \in \mathbb{R}^n$ is the state
- $\boldsymbol{y}_k \in \mathbb{R}^m$ is the output
- $\boldsymbol{u}_k \in \mathbb{R}^r$ is the input
- $\boldsymbol{d}_k \in \mathbb{R}^q$ is the unknown input
- $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are independent zero-mean white noise sequences with the covariance matrices $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$

➜ Unknown input observer

$$\boldsymbol{z}_{k+1} = \boldsymbol{F}_{k+1}\boldsymbol{z}_k + \boldsymbol{T}_{k+1}\boldsymbol{B}_k\boldsymbol{u}_k + \boldsymbol{K}_{k+1}\boldsymbol{y}_k$$
$$\hat{\boldsymbol{x}}_{k+1} = \boldsymbol{z}_{k+1} + \boldsymbol{H}_{k+1}\boldsymbol{y}_{k+1}$$

➜ Alternative form of the UIO

$$\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{x}}_{k+1/k} + \boldsymbol{H}_{k+1}\boldsymbol{\varepsilon}_{k+1/k} + \boldsymbol{K}_{1,k+1}\boldsymbol{\varepsilon}_k,$$

where

$$\hat{\boldsymbol{x}}_{k+1/k} = \boldsymbol{A}_k\hat{\boldsymbol{x}}_k + \boldsymbol{B}_k\boldsymbol{u}_k$$
$$\boldsymbol{\varepsilon}_{k+1/k} = \boldsymbol{y}_{k+1} - \hat{\boldsymbol{y}}_{k+1/k} = \boldsymbol{y}_{k+1} - \boldsymbol{C}_{k+1}\hat{\boldsymbol{x}}_{k+1/k}$$
$$\boldsymbol{\varepsilon}_k = \boldsymbol{y}_k - \hat{\boldsymbol{y}}_k$$

➜ Unknown input observer

The necessary condition for the existence of a solution to the unknown input de-coupling problem is (Chen and Patton, 1999)

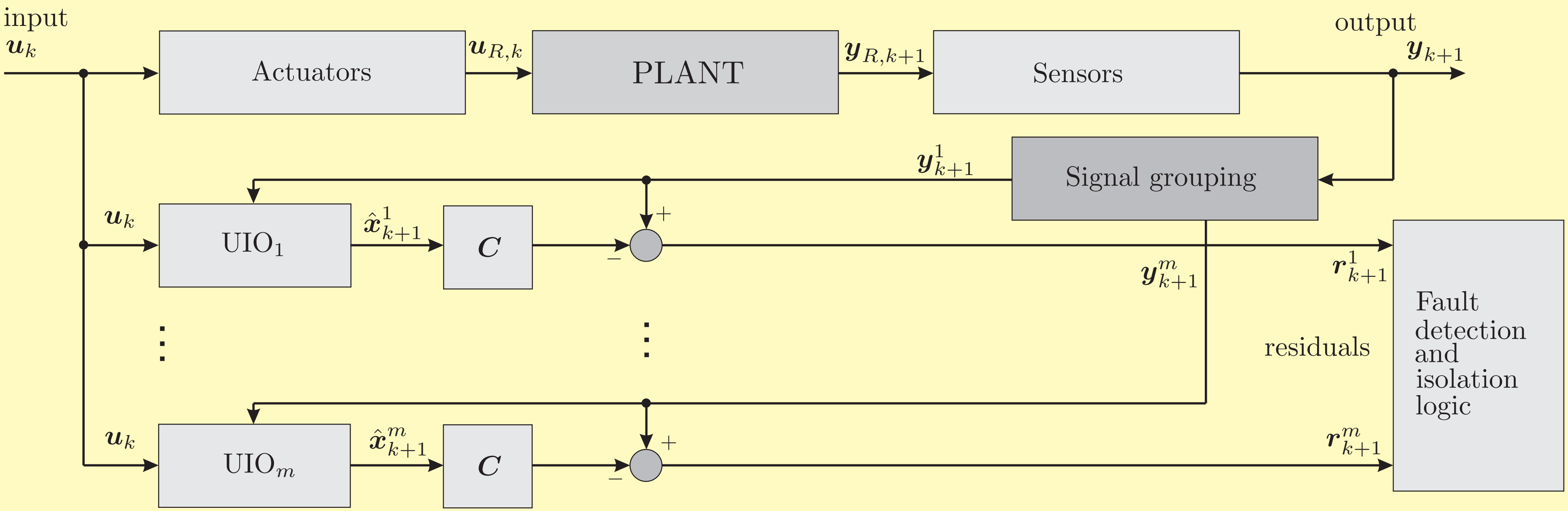
$$\mathrm{rank}(\boldsymbol{C}_{k+1}\boldsymbol{E}_k) = \mathrm{rank}(\boldsymbol{E}_k)$$

and a special solution is

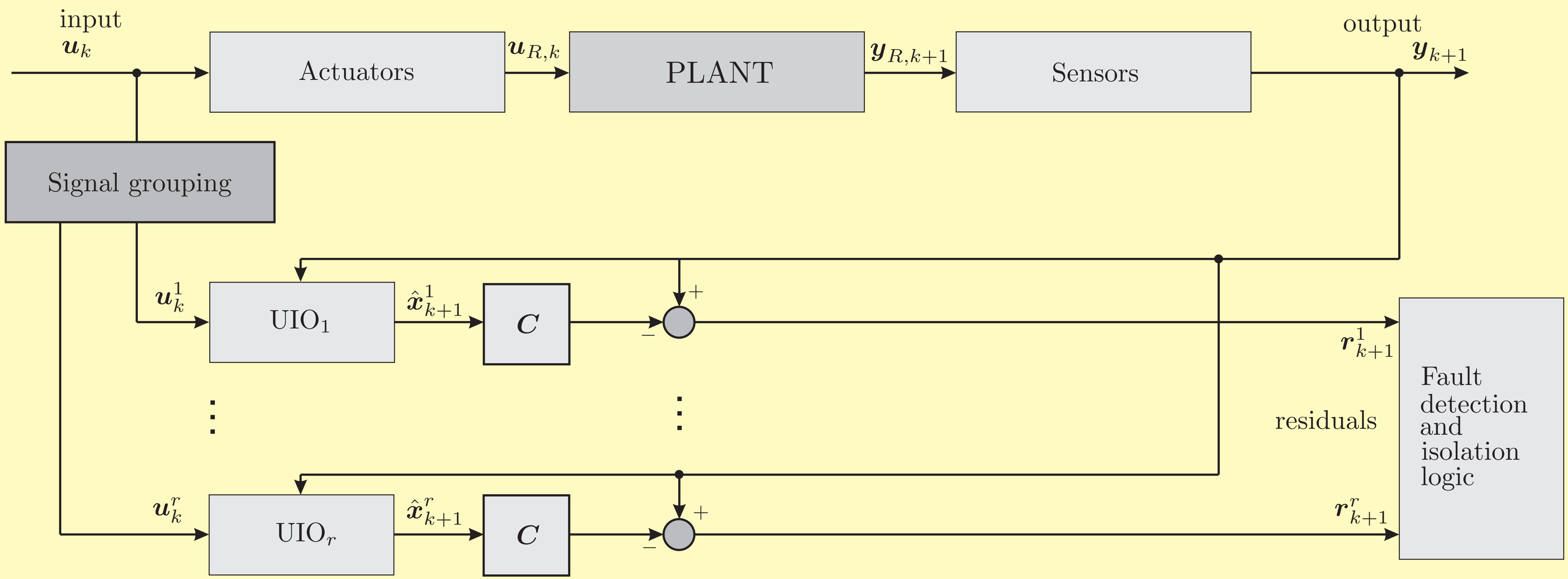
$$\boldsymbol{H}^*_{k+1} = \boldsymbol{E}_k \left[(\boldsymbol{C}_{k+1}\boldsymbol{E}_k)^T \boldsymbol{C}_{k+1}\boldsymbol{E}_k\right]^{-1} (\boldsymbol{C}_{k+1}\boldsymbol{E}_k)^T$$

The above solution makes it possible to de-couple the unknown input from the state estimation error and, as a consequence, from the residual.

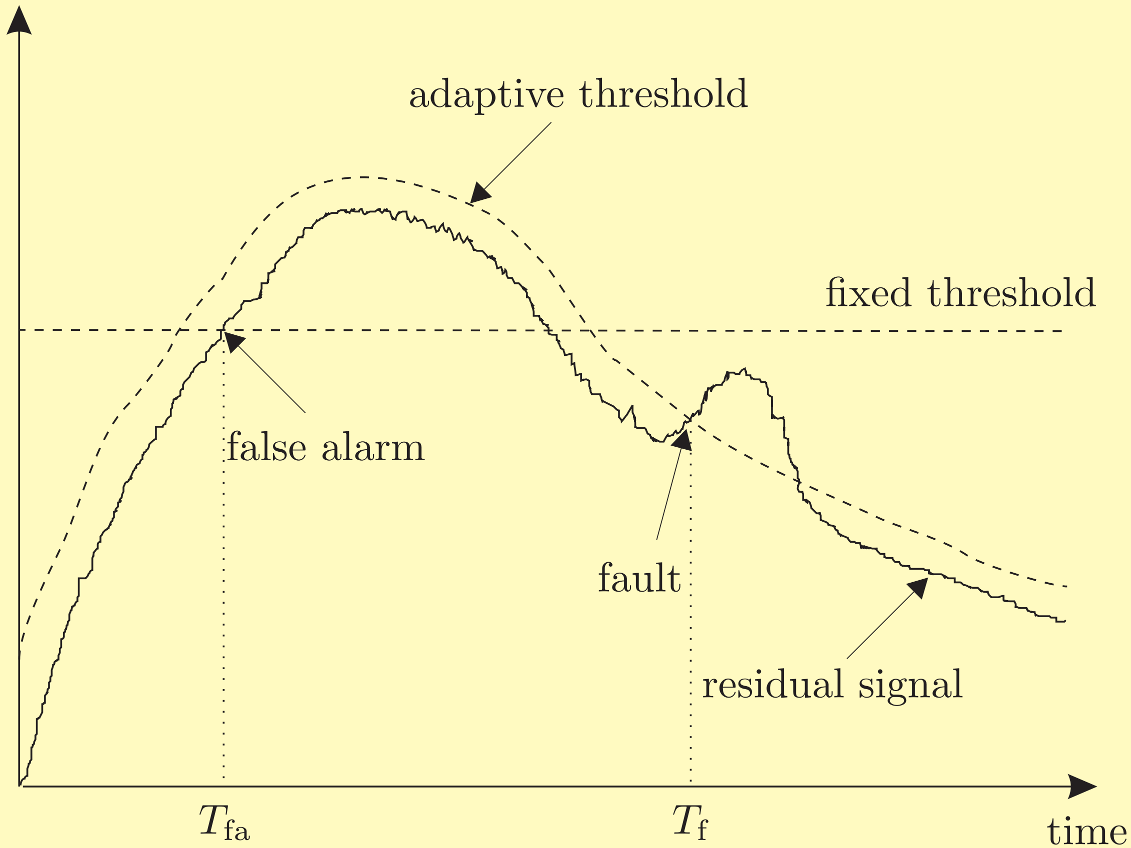
➜ Unknown input observer – sensor FDI scheme



➜ Unknown input observer – actuator FDI scheme



➜ Passive approaches – adaptive threshold



➜ Adaptive threshold in parameter-estimation-based fault diagnosis schemes

- The system is linear-in-parameters and can be described as follows:

$$\boldsymbol{y}_k = \boldsymbol{z}_k^T \boldsymbol{p} + \boldsymbol{\varepsilon}_k$$

where $\boldsymbol{z}_k$ stands for the regressor vector and $\boldsymbol{\varepsilon}_k$ denotes the noise.

- A recursive least-square technique is employed for parameter estimation:

$$\begin{aligned}
\hat{\boldsymbol{p}}_k &= \hat{\boldsymbol{p}}_{k-1} + \boldsymbol{k}_k \varepsilon_k \\
\boldsymbol{k}_k &= \boldsymbol{P}_{k-1}\boldsymbol{z}_k \left(1 + \boldsymbol{z}_k^T \boldsymbol{P}_{k-1}\boldsymbol{z}_k\right)^{-1} \\
\varepsilon_k &= y_k - \boldsymbol{z}_k^T \hat{\boldsymbol{p}}_k \\
\boldsymbol{P}_k &= \left[\boldsymbol{I}_{n_p} - \boldsymbol{k}_k \boldsymbol{z}_k^T\right] \boldsymbol{P}_{k-1}
\end{aligned}$$

Residual $\boldsymbol{r}_k = \boldsymbol{p}_0 - \hat{\boldsymbol{p}}_k$ and its adaptive threshold (at α-level):

$$|r_{i,k}| < t_\alpha \hat{\sigma} \sqrt{s_{i,k}}, \quad i = 1, \dots, n_p,$$

where t_α is the t-Student distribution quantile, $\boldsymbol{s}_k = \mathrm{diag}(\boldsymbol{P}_k)$, $\hat{\sigma}$ is the noise standard deviation estimate

➜ Adaptive threshold in input-output fault diagnosis schemes

Main assumptions and concepts: Emami-Naeini *et al.* (1988); Ding and Frank (1991)

- The residual can be described in the frequency domain

$$r(s) = H(s)G_f(s)f(s) + H(s)\Delta G_u(s)u(s)$$

  where

  – $H(s)$ represents system dynamics

  – $G_f(s)$ describes the influence of faults $f(s)$ on the system

  – $\Delta G_u(s)$ denotes model uncertainty

  – $u(s)$ is. the input

- Model uncertainty is bounded:

$$\|\Delta G_u(s)\| < \delta$$

➜ Adaptive threshold in input-output fault diagnosis schemes

Main assumptions and concepts:

- Fault-free residual

$$r(s) = H(s)\Delta G_u(s)u(s)$$

  and its norm:

$$\|r(s)\| = \|H(s)\Delta G_u(s)u(s)\| \leqslant \|H(s)u(s)\|\|\Delta G_u(s)\| \leqslant \delta\|H(s)u(s)\|$$

- The adaptive threshold is generated by the system of the form

$$T(s) = \delta H(s)u(s)$$

- The fault detection logic boils down to checking

$$\|r(t)\| > \|T(t)\|$$

- An optimization procedure can be implemented in order to increase sensitivity to faults

☛ TACKLING NON-LINEARITIES AND THE ROBUSTNESS PROBLEM

➜ Unknown Input Observers for non-linear systems – deterministic systems

- Extended UIO: Witczak *et al.* (2002)
- UIO for Lipschitz non-linear systems: Koenig and Mammar (2001); Witczak (2005)
- UIO for polynomial and binomial systems: Shields (2001)
- UIO – coordinate transformation: Seliger and Frank (2000)

➜ Unknown input observers for non-linear systems – stochastic systems

- Extended UIO: Witczak *et al.* (2002)
- UIO particle filter – no work reported

☛ ISSUES OF ANALYTICAL TECHNIQUES = CHALLENGES FOR SOFT COMPUTING

❒ Difficulties in developing non-linear models:

- there is no general analytical framework for non-linear system identification
- the parameter estimation problem is often a global optimization task

❒ Issues in designing fault diagnosis schemes:

- insensitivity to (noise+disturbances+unknown inputs) + sensitivity to faults = global and multi-objective optimization problems
- designing non-linear observers: increasing convergence rate + robustness to model uncertainty = global structure optimization task

❒ Alternative methods of model uncertainty representation